

Theorem 1.2 (Tail Sum Formula). *Let X be a random variable that only takes on values in \mathbb{N} . Then*

$$E(X) = \sum_{k=1}^{\infty} \Pr(X \geq k)$$

Proof. We manipulate the formula for the expectation:

$$\begin{aligned} E(X) &= \sum_{x=1}^{\infty} x \Pr(X = x) \\ &= \sum_{x=1}^{\infty} \sum_{k=1}^x \Pr(X = x) \\ &= \sum_{k=1}^{\infty} \sum_{x=k}^{\infty} \Pr(X = x) \\ &= \sum_{k=1}^{\infty} \Pr(X \geq k) \end{aligned}$$

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