Definition [edit]

In short, a probability space is a measure space such that the measure of the whole space is equal to one.

The expanded definition is the following: a probability space is a triple $(\Omega, \mathcal{F}, P)$ consisting of:

- the sample space Ω — an arbitrary non-empty set,
- the σ-algebra $\mathcal{F} \subseteq 2^\Omega$ (also called σ-field) — a set of subsets of Ω, called events, such that:
  - $\mathcal{F}$ contains the sample space: $\Omega \in \mathcal{F}$,
  - $\mathcal{F}$ is closed under complements: if $A \in \mathcal{F}$, then also $(\Omega \setminus A) \in \mathcal{F}$,
  - $\mathcal{F}$ is closed under countable unions: if $A_i \in \mathcal{F}$ for $i = 1, 2, \ldots$, then also $(\bigcup_{i=1}^\infty A_i) \in \mathcal{F}$
    - The corollary from the previous two properties and De Morgan's law is that $\mathcal{F}$ is also closed under countable intersections: if $A_i \in \mathcal{F}$ for $i = 1, 2, \ldots$, then also $(\bigcap_{i=1}^\infty A_i) \in \mathcal{F}$
- the probability measure $P : \mathcal{F} \to [0, 1]$ — a function on $\mathcal{F}$ such that:
  - P is countably additive (also called σ-additive): if $\{A_i\}_{i=1}^\infty \subseteq \mathcal{F}$ is a countable collection of pairwise disjoint sets, then $P(\bigcup_{i=1}^\infty A_i) = \sum_{i=1}^\infty P(A_i)$,
  - the measure of entire sample space is equal to one: $P(\Omega) = 1$.