If x_n is not Cauchy then an $\varepsilon > 0$ can be chosen (fixed in the rest) for which, given any arbitrarily large N there are $p, q \geq n$ for which $p < q$ and $x_q - x_p > \varepsilon$.

Now start with $N = 1$ and choose $x_{n_1},\ x_{n_2}$ for which the difference of these is at least ε. Next use some N' beyond either index $n_1,\ n_2$ and pick $N' < n_3 < n_4$ for which $x_{n_4} - x_{n_3} > \varepsilon$. Continue in this way to construct a subsequence.

That this subsequence diverges to $+\infty$ can be shown using the Archimedes principle, which you say can be used, since all the differences are nonnegative and there are infinitely many differences each greater than ε, a fixed positive number.