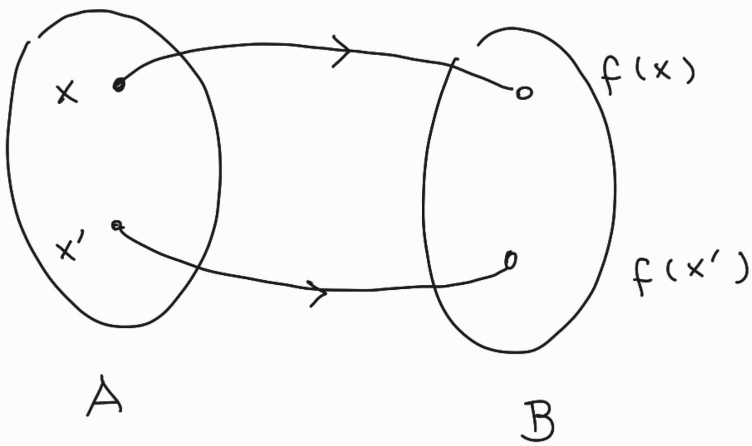
Definition (Injective, one-to-one)

$f : A \to B$ is injective or one-to-one if for every $x \in A$ and $x' \in A$ with $x' \neq x$ we have $f(x) \neq f(x')$.



In other words, two distinct elements in A do not have the same image.

Examples:

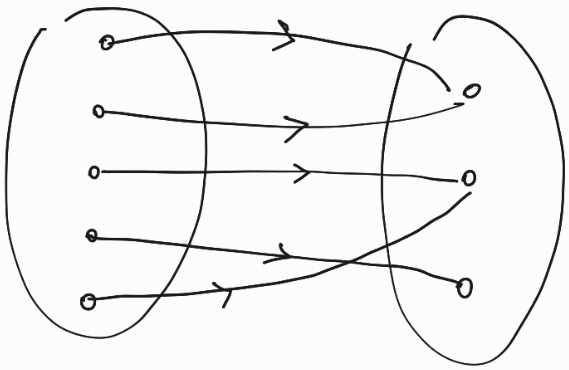
$f : \mathbb{R} \to \mathbb{R}$, $f(x) = x$ is injective

$g : \mathbb{R} \to \mathbb{R}$, $g(x) = x^2$ is not injective

Definition (Surjective, onto)

$f : A \to B$ is a surjective or onto function if for $b \in B$ there exists $a \in A$ such that $f(a) = b$.

In other words, $\text{Image}(f) = B$.



A B

Examples:

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = 3x + 2$ is surjective

$g : \mathbb{R} \to \mathbb{R}$, $g(x) = x^2$ is not surjective.

$h : \mathbb{Z} \to \mathbb{Z}$, $h(x) = 3x+1$ is not surjective.

We cannot have $h(x) = 2$ for some $x \in \mathbb{Z}$, as the equation $3x+1 = 2$ has no solutions in the integers.

<u>Definition</u> (bijection)

$f : A \to B$ is a bijection if f is injective and surjective.

<u>Examples</u>:

$f : \mathbb{R} \to \mathbb{R}$

$f: \mathbb{R} \to \mathbb{R}$, $f(x) = 2x+1$ is a bijection

$g: \mathbb{R} \to \mathbb{R}$, $g(x) = (x-1)(x+1)$ is not a bijection.

<u>Definition</u> (Identity function)

The identity function on a set S is a function $f: S \to S$ defined by $f(x) = x \quad \forall x \in S$.

<u>Composition of functions:</u>

Let $f: A \to B$ and $g: C \to A$ be functions.

The function $f \circ g : C \to B$ defined by $(f \circ g)(x) = f(g(x))$ is the composition of f with g.

<u>Examples:</u>

$f: \mathbb{R} \to \mathbb{R}$, $f(x) = 2x+1$

$g: \mathbb{R} \to \mathbb{R}$, $g(x) = 6x+7$

$h: \mathbb{R} \to \mathbb{R}$, $h(x) = x^2$

We have: $f(g(x)) = f(6x+7) = 2(6x+7)+1$

$$= 12x + 15$$

$$f(h(x)) = f(x^2) = 2x^2+1$$

$$h(f(x)) = h(2x+1) = (2x+1)^2$$

$$= 4x^2 + 4x + 1$$

Note that $h \circ f \neq f \circ h$ in this example.

Definition (Inverse function)

Let $f : A \to B$ be a bijective function. The inverse function of f is the unique function $g : B \to A$ such that

$$g(f(x)) = x \text{ for all } x \in A$$

and

$$f(g(y)) = y \text{ for all } y \in A.$$

Notation: f^{-1}.

Examples:

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = x$: $f^{-1}(x) = x$

$g : \mathbb{R} \to \mathbb{R}$, $g(x) = 2x + 1$:

$$x = g(g^{-1}(x)) = 2g^{-1}(x) + 1$$

This gives $\dfrac{x-1}{2} = g^{-1}(x)$.

Definition (Square-root function)

The square-root function is the inverse

The square-root function is the inverse function of $f : [0,\infty) \to [0,\infty)$, $f(x) = x^2$.

In particular, $\sqrt{(-3)^2} = -3$ is FALSE.

Definition (Floor function)

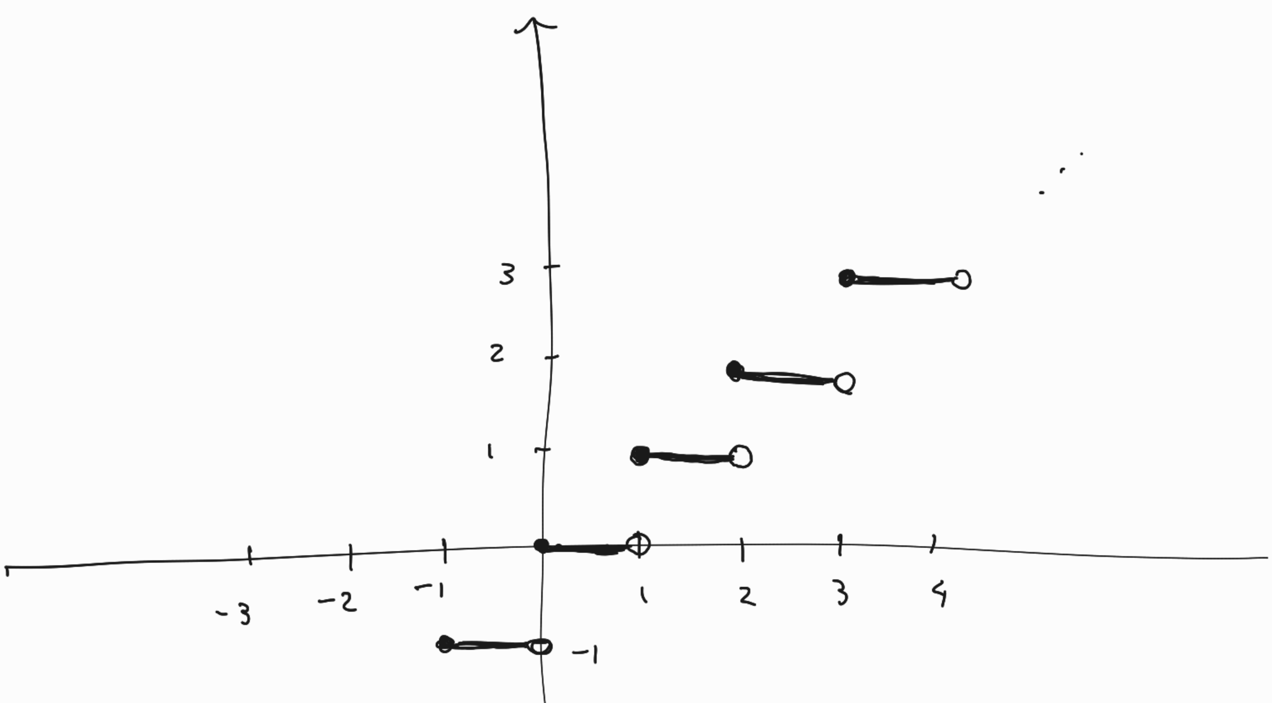
The floor of a number x is the largest integer smaller or equal to x. Notation : $\lfloor x \rfloor$.
Therefore, if $i \in \mathbb{Z}$ and $i \le x < i+1$, then

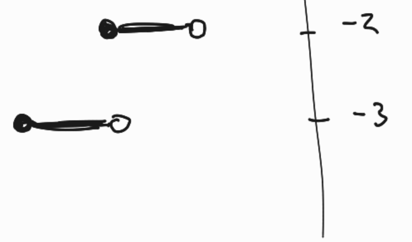
$$\lfloor x \rfloor = i.$$

Example :

- $\lfloor 3.99 \rfloor = 3$ • $\lfloor 1 \rfloor = 1$ • $\lfloor 0 \rfloor = 0$ • $\lfloor -1.5 \rfloor = -2$

The graph of the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \lfloor x \rfloor$:





<u>Definition</u> (Ceiling function)

The ceiling of a number x is the smallest integer greater or equal to x. <u>Notation</u>: $\lceil x \rceil$.

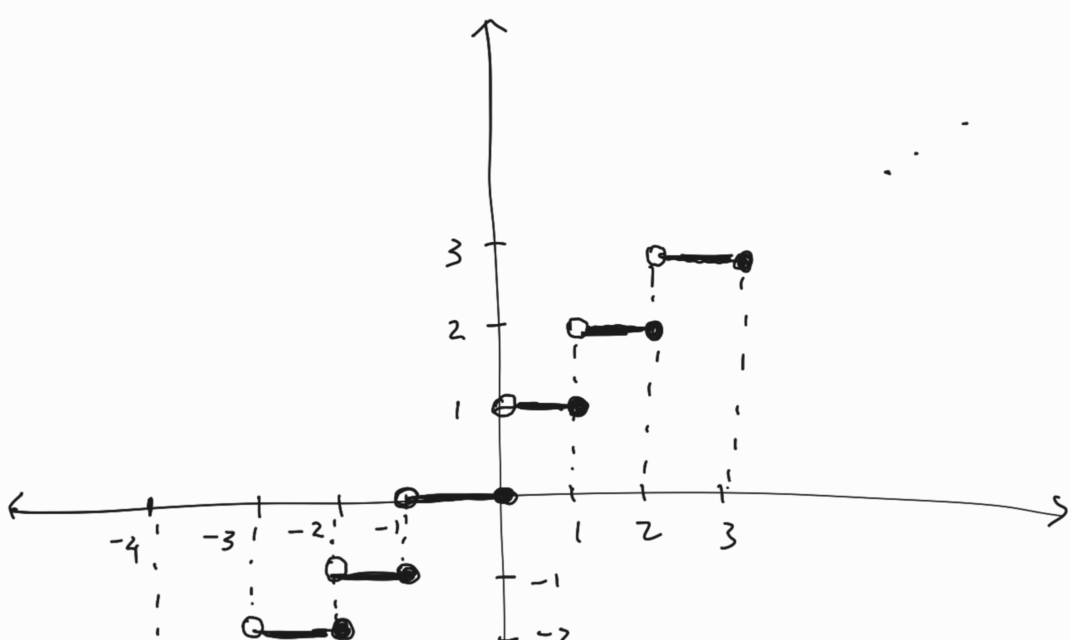
Therefore, if $i \in \mathbb{Z}$ and $i-1 < x \leq i$, then

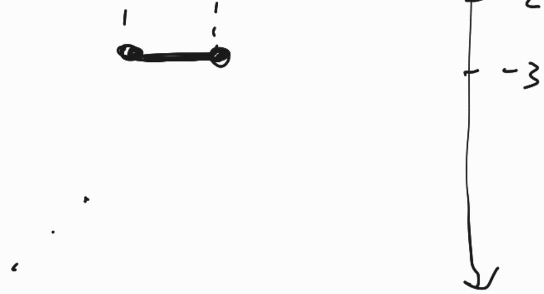
$$\lceil x \rceil = i .$$

<u>Example</u>:

- $\lceil 3.99 \rceil = 4$ • $\lceil 1 \rceil = 1$ • $\lceil 0 \rceil = 0$ • $\lceil -1.5 \rceil = -1$

<u>The graph of the function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \lceil x \rceil$:</u>

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